Entanglement and teleportation of macroscopic continuous variables by superconducting devices

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A current-biased low-temperature superconducting Josephson junction (JJ) is dynamically describable by the quantized motion of a fictitious particle in a "washboard" potential[1]. The long coherence time of tightly-bound states in the washboard potential of a JJ has prompted the effort[2−6] to couple JJs and operate them as entangled qubits, capable of forming building blocks of a scalable quantum computer. Here we consider a hitherto unexplored quantum aspect of coupled JJs: the ability to produce Einstein-Podolsky-Rosen (EPR) entanglement of their continuous variables[7], namely, their magnetic fluxes and induced charges. Such entanglement, apart from its conceptual novelty, is the prerequisite for a far-reaching goal: teleportation[8−11] of the flux and charge variables between JJs, implementing the transfer of an unknown quantum state along a network of such devices.

For our analysis, we shall adopt the following Hamiltonian[5] of two JJs coupled by capacitance C_c, written in terms of the quantized variables $\theta_{1,2} = \Phi_{1,2}/\Phi_0$, i.e., their magnetic fluxes normalized to the flux quantum $\Phi_0 = 2\pi\hbar/(2e)$, and their canonically-conjugate variables $p_{1,2} = -i\partial/(\partial\theta_{1,2})$, related to induced charges:

$$\begin{aligned} H &= 4E_C(1+\zeta)^{-1}(p_1^2+p_2^2+2\zeta p_1 p_2) \\ &\quad -E_J(\cos\theta_1+\cos\theta_2+J_1\theta_1+J_2\theta_2). \end{aligned} \tag{1}$$

Here $E_C = e^2/C_J$, C_J being the single-junction capacitance; $E_J = \hbar I_c/(2e)$, I_c being the critical current; $J_{1,2} = I_{1,2}/I_c$ are the normalized bias currents, and $\zeta = C_c/(C_c+C_J)$ is the coupling parameter.

In order to study EPR correlations, we adopt the basis of collective variables

$$p_\pm = (p_1 \pm p_2)/\sqrt{2}, \quad \theta_\pm = (\theta_1 \pm \theta_2)/\sqrt{2}, \tag{2}$$

thereby rewriting Eq. (1) as

$$\begin{aligned} H &= \frac{p_+^2}{2m_+} + \frac{p_-^2}{2m_-} - \frac{E_J}{\sqrt{2}}(J_1+J_2)\theta_+ \\ &\quad -\frac{E_J}{\sqrt{2}}(J_1-J_2)\theta_- - 2E_J\cos\frac{\theta_+}{\sqrt{2}}\cos\frac{\theta_-}{\sqrt{2}}. \end{aligned} \tag{3}$$

Here the + and − collective modes are characterized by different "masses" $m_\pm = (1+\zeta)/[8E_C(1\pm\zeta)]$. The coupling of θ_+ and θ_- via the last term in Eq. (3) renders the dynamics two-dimensional (2D) in the + and − modes.

EPR correlations occur only between two *commuting* variables, e.g., θ_+ and p_- or θ_- and p_+[7,8,12]. Ideal EPR correlations require these variables to be dynamically decoupled[7,8,11,12]. Such decoupling of the + and − modes holds when the Born-Oppenheimer (BO) approximation[13], which is widely used in molecular dynamics, is valid. To impose the BO approximation, we draw a parallel between the − and + modes of the coupled JJs, and, respectively, slow (nuclear) and fast (electronic) degrees of freedom of a molecule. This parallel holds for a large difference of the corresponding masses

$$m_+ \ll m_- . \tag{4}$$

Condition (4), which justifies the BO decoupling of the fast and slow modes[13], means that, during the oscillation of the θ_--mode, the θ_+-mode always remains on the same potential curve. Mathematically, the BO approximation implies that the eigenfunctions of Eq. (3) are factorizable as $\varphi(\theta_+)\Phi(\theta_-)$, with the fast-mode eigenfunctions $\{\varphi_n(\theta_+)\}$ and the corresponding energy eigenvalues $\{\varepsilon_n\}$, $n = 0, 1, 2, \ldots$, being parametrically dependent on the slow-mode coordinate θ_-.

Let us first consider the unbiased case, $J_{1,2} = 0$. Then, to zeroth order in m_+/m_-, the bound-state energies $\varepsilon(\theta_-^{(0)})$ of the fast mode are minimized for $\theta_-^{(0)} = 0$. They are the Mathieu (pendulum) equation eigenvalues, whose form for $E_J \gg 1/m_+ = 8E_C$ (the "phase regime"[2−6]) is weakly anharmonic:

$$\begin{aligned} \varepsilon_n(0) &\approx -2E_J + \hbar\omega_0\left(n+\frac{1}{2}\right) - \frac{1}{16m_+}\left(n^2+n+\frac{1}{2}\right), \\ \hbar\omega_0 &= \sqrt{E_J/m_+}. \end{aligned} \tag{5}$$

The bound-state solutions for the slow mode, to first order in $\sqrt{m_+/m_-}$, are obtained upon retaining quadratic (harmonic) terms. The corresponding slow-mode energies, specified by two quantum numbers (n and ν), are

$$E_{n,\nu} \approx \hbar\Omega_n\,(\nu+1/2), \tag{6}$$

$$\Omega_n \approx \omega_0\left(1-\frac{n+1/2}{4\sqrt{E_J m_+}}\right)\sqrt{\frac{m_+}{m_-}}. \tag{7}$$

In the weakly-biased case, $|J_{1,2}| \ll 1$, the ratio of the level spacings of the slow and fast modes is also of the order of $\sqrt{m_+/m_-}$, but the excited levels are somewhat broadened by leakage to the continuum, [Fig. 1(a,b)].

For the states described by Eqs. (5 − 7), the signature of EPR correlations (entanglement) is twofold: (a) The product of *commuting-variable* variances of the collective (+ and −) modes is reduced well below the Heisenberg uncertainty limit[14]

$$\sqrt{\langle\Delta\theta_\pm^2\rangle_{n,\nu}\langle\Delta p_\mp^2\rangle_{n,\nu}} \equiv \frac{1}{2s} \ll \frac{1}{2}, \tag{8}$$

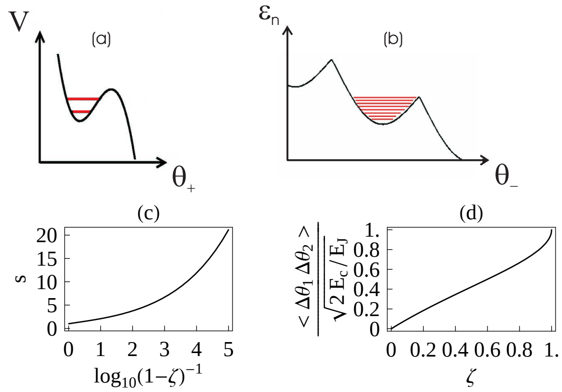

FIG. 1: (a) Fast-mode washboard potential with few bound levels, large tunneling rate for excited states, (b) slow-mode washboard potential with many energy levels and possibility of *quasiclassical* motion, (c) squeezing s and (d) normalized cross-correlation $(E_j/(2E_c))^{1/2}\langle\Delta\theta_1\Delta\theta_2\rangle$ for the ground state, plotted versus $\log_{10}(1-\zeta)^{-1}$ (in (c)) or coupling ζ (in (d)).

implying that the variables θ_- and p_+ or their conjugates become "squeezed"[14]. Appreciable "squeezing", $s > 3$, is achievable for $\zeta > 0.976$. (b) The cross-terms $\langle\Delta\theta_1\Delta\theta_2\rangle_{n,\nu}$ and $\langle\Delta p_1\Delta p_2\rangle_{n,\nu}$ in $\langle\Delta\theta_-^2\rangle_{n,\nu}$ and $\langle\Delta p_+^2\rangle_{n,\nu}$, expressing inter-junction voltage and current correlations, are nonvanishing and tend to grow in absolute value with the coupling parameter ζ [Fig. 1(c)].

In order to prepare these EPR-correlated states, we may start from uncoupled JJ1 and JJ2 ($\zeta = 0$), both in the ground state, and quasi-adiabatically increase ζ. The quasiadiabaticity confines θ_+ and θ_- to their respective ground states, but the ground-state uncertainties in Eq. (8) become progressively "squeezed". Quasiadiabatic change of the coupling capacitance C_c and, hence, of ζ may be attained by gradually switching-on a battery of ferroelectric capacitors [15] connected in parallel, much slower than Ω_0^{-1} but well within the JJ coherence time.

If indeed Eq. (8) is satisfied for a certain interaction time, as in Fig. 1(c), we may use these EPR correlations for implementing the continuous-variable teleportation protocol[11]. To this end, consider JJ3 to be uncoupled from the EPR-correlated JJs 1 and 2. We wish to teleport the unknown state $|\psi_3\rangle$ of JJ3 to the distant JJ1. The crucial step of the protocol is the measurement of the *joint variables*

$$\theta_{23}^{(-)} = \theta_2 - \theta_3, \quad p_{23}^{(+)} = p_2 + p_3. \tag{9}$$

This can be effected by *classical probes* that are sensitive to the sum of fluxes (currents) and the difference of voltages of JJ2 and JJ3.

The next step is the communication of the measurements' results $\bar{\theta}_{23}^{(-)}$ and $\bar{p}_{23}^{(+)}$ to the "controller" of JJ1, who then switches off the coupling ζ of JJ1 and JJ2 and "shifts" the flux and voltage of JJ1:

$$\theta_1 \to \theta_1 - \bar{\theta}_{23}^{(-)}, \quad p_1 \to p_1 + \bar{p}_{23}^{(+)}. \tag{10}$$

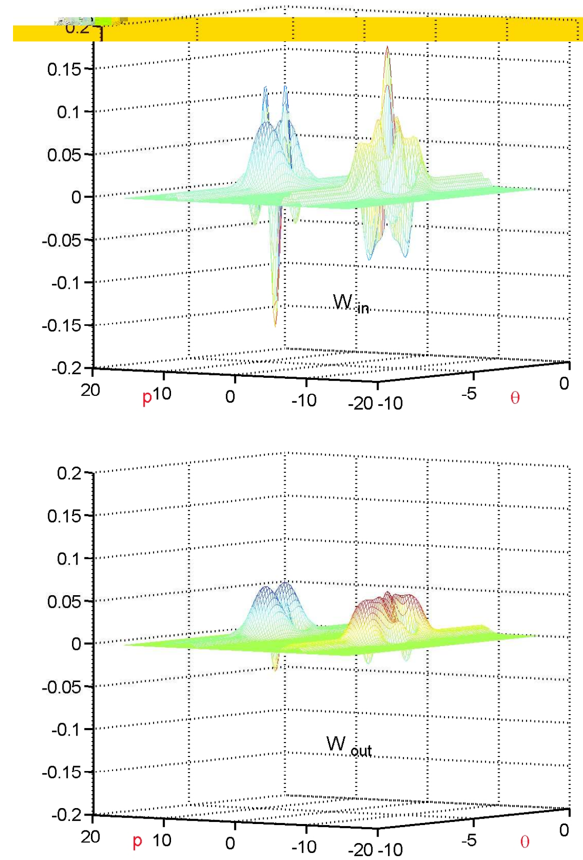

FIG. 2: (a) Wigner function for the input states $(|1\rangle - i|3\rangle)/\sqrt{2}$ (left) and $(|0\rangle - i|2\rangle + i|4\rangle)/\sqrt{3}$ (right), and (b) corresponding output (teleported) states, for an effective coupling $\zeta = 0.9995$ (as per Eq. (11))

Provided that the protocol has been implemented with sufficiently high fidelity, it transforms the state of JJ1 to replicate $|\psi_3\rangle$ (see[11]), yielding $\theta_1 \to \theta_3$, $p_1 \to p_3$. In Fig. 2 we demonstrate the possibility of high-fidelity teleportation of a superposition (wavepacket) consisting of either 3 or 2 "washboard"-potential eigenfunctions $\psi_3(\theta) = \sum c_j(0)\varphi_j(\theta)$ from JJ3 [Fig. 2(a)] to JJ1 [Fig. 2(b)].

The uncertainty ("noise") that degrades the fidelity of the teleportation protocol is determined by[11,12]

$$\begin{aligned}\langle\Delta\theta_T^2\rangle &= \langle\Delta\theta_{12}^{(-)\,2}\rangle + \langle\Delta\theta_{23}^{(-)\,2}\rangle,\\ \langle\Delta p_T^2\rangle &= \langle\Delta p_{12}^{(+)\,2}\rangle + \langle\Delta p_{23}^{(+)\,2}\rangle,\end{aligned} \tag{11}$$

where $\langle\Delta\theta_{12}^{(-)\,2}\rangle$ and $\langle\Delta p_{12}^{(+)\,2}\rangle$ are determined by the EPR correlations discussed above, while $\langle\Delta\theta_{23}^{(-)\,2}\rangle$ and $\langle\Delta p_{23}^{(+)\,2}\rangle$ depend on the accuracy of measuring the joint variables $\theta_{23}^{(-)}$ and $p_{23}^{(+)}$ and the transfer (shift) accuracy. High fidelity (defined as the overlap of the input and output states) requires that $\langle\Delta\theta_T^2\rangle\langle\Delta p_T^2\rangle \ll 1$. For the conceivable parameters[6] chosen in Fig. 2(b), this "noise"

entails $\langle \Delta\theta_T^2 \rangle \simeq 0.015$, $\langle \Delta p_T^2 \rangle \simeq 3.19$, which limits the fidelity of the protocol to $\lesssim 0.82$.

To conclude, we have demonstrated the feasibility, in principle, of EPR entanglement and teleportation of macroscopic continuous variables in superconducting Josephson junctions. Beyond the innovation of performing quantum operations upon such macroscopic observables as magnetic flux and induced voltage, their realization would allow the development of quantum-information transfer networks comprised of superconducting elements. Such transfer may be used to interface blocks of quantum-information processors.

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